

On the GUT scale of F-Theory $SU(5)$

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Abstract

In F-theory GUTs, threshold corrections from Kaluza-Klein (KK) massive modes arising from gauge and matter multiplets play an important role in the determination of the weak mixing angle and the strong gauge coupling of the effective low energy model. In this letter we further explore the induced modifications on the gauge couplings running and the GUT scale. In particular, we focus on the KK-contributions from matter curves and analyse the conditions on the chiral and Higgs matter spectrum which imply a GUT scale consistent with the minimal unification scenario. As an application, we present an explicit computation of these thresholds for matter fields residing on specific non-trivial Riemann surfaces.

1 Introduction

It is well known that the spectrum of the minimal supersymmetric extension of the Standard Model (SM) is consistent with a gauge couplings unification at a scale $M_{GUT} \sim 2 \times 10^{16}$ GeV. This fact corroborates the point of view that the SM gauge group factors emanate from a higher unified gauge symmetry. In the simplest case, the SM gauge symmetry is embedded in the $SU(5)$ Grand unified Theory (GUT) while the SM matter content is nicely assembled into $SU(5)$ multiplets. In addition, although string theory appears to be the appropriate candidate for incorporating gravity into the unification scenario, one must still confront the mismatch between M_{GUT} and the natural gravitational scale $M_{Pl} \sim 1.2 \times 10^{19}$ GeV. Thus, a plausible implementation of unification, requires a string theory formulation in which the gauge theory decouples from gravity at the desired scale.

Recently, there have been considerable efforts to develop a viable effective field theory model from F-theory [1]¹. This picture consists of a 7-brane wrapping a compact Kähler surface S of two complex dimensions while the gauge theory of a particular model is associated with the geometric singularity of the internal space [5, 6, 7, 8]. In this set up it is possible to decouple gauge dynamics from gravity by restricting to compact surfaces S that are of del Pezzo type. The exact determination of the GUT scale however, may depend on the spectrum and other details of the chosen gauge symmetry and on the particular model. In the present work, we will assume the minimal unified $SU(5)$ GUT.

A reliable computation of the GUT scale should also take into consideration the various threshold corrections. These may also depend on the choice of the specific compactification as well as the particular model. In F-theory $SU(5)$ we are examining here, there are several sources of threshold effects that have to be taken into account [10, 11, 12, 13, 14, 15]. Thus, we encounter thresholds related to the flux mechanism (used to break the GUT gauge symmetry) which induce splitting of the gauge couplings at the GUT scale [10, 11]. A second source concerns threshold corrections generated from heavy KK massive modes [10, 14]. Furthermore, corrections to gauge coupling running arise due to the appearance of probe D3-branes generically present in F-theory compactifications and filling the $3 + 1$ non-compact dimensions while sitting at certain points of the internal manifold [15]. Finally, threshold effects are generated at scales $\mu < M_{GUT}$ when additional light degrees of freedom and in particular superpartners are integrated out. The effects of the latter have been extensively studied in the context of supersymmetric and String Grand Unified Theories². In reasonable circumstances, (for example when no-extra degrees of freedom remain below M_{GUT}) the last two categories can be made consistent with two loop corrections and a unification scale of the order of $M_{GUT} \sim 2 \times 10^{16}$ GeV.

Thresholds induced by the flux mechanism have been extensively analyzed in recent literature [10, 11, 13]. There, it was shown that the $U(1)_Y$ -flux induced splitting is compatible with the GUT embedding of the minimal supersymmetric standard model,

¹For comprehensive reviews see [2, 3, 4]

²For an incomplete list see [16].

provided that no extra matter other than color triplets is present in the spectrum. Thresholds originating from KK-massive modes have been discussed in [10] and were found to be related to a topologically invariant quantity, the Ray-Singer analytic torsion [17]. This observation was originally made for the case of manifolds with G_2 holonomy where thresholds were computed and estimates for the GUT scale were given [18]. In the case of F-theory however, the situation is a little more complicated. Indeed, in M-theory one assumes that massless $SU(5)$ multiplets are generated at singularities of the internal space which are believed to be conical [18]. Since conical singularities induce no new length, it is expected that no new massive particles are introduced. On the contrary, in F-theory, KK-massive modes exist for both the gauge and the matter fields. To be more precise, in the present context of the $SU(5)$ theory, these come along with massless gauge fields propagating in the bulk, while the chiral matter as well as the Higgs representations reside on two-dimensional Riemann surfaces (matter curves). In general, both kinds of KK-modes contribute to the gauge coupling running and can in principle modify the unification scale. It is straightforward to estimate the modification induced by the vector supermultiplet, nevertheless the contributions of the matter fields might be model dependent. In this letter we aim to revisit this second source of threshold corrections. We will discuss this issue in the context of models where chiral matter and Higgs fields occupy complete $SU(5)$ multiplets. We will show that under reasonable assumptions for the matter curve bundle structure, no further modifications are induced from the corresponding matter KK-massive modes.

To make the presentation self-contained, we will first briefly review the eight-dimensional twisted theory and obtain the degrees of freedom together with their corresponding topological properties. In section 3 we will compute the threshold corrections. After recapitulating the basic steps for the gauge contributions, we will continue with a detailed determination of the thresholds from the matter curves. Next, we will proceed with an explicit calculation of the KK-massive modes thresholds originating from chiral and Higgs matter curves and show that their only net effect amounts to a shift of the common gauge coupling at the GUT scale. In section 4 we will present our conclusions.

2 Twisted gauge theory and degrees of freedom

Before proceeding to the computation of the threshold corrections and following [7, 8], we will first review the salient features of the theory and summarize the properties of the massless and massive degrees of freedom respectively. F-theory is defined locally by the worldvolume of 7-branes of ADE-type singularity which for definiteness we assume to be $SU(5)$. We will further assume that a $U(1)_Y$ flux is turned on on the 7-brane in order to break $SU(5)$ down to the Standard Model (SM). We consider a maximally Supersymmetric Yang-Mills (YM) theory in 10 dimensions on $R^{9,1}$ with field content consisting of a ten dimensional vector \mathcal{A}_I , ($I = 0, 1, \dots, 9$) and an adjoint valued fermion transforming under $SO(9, 1)$ as a positive chirality spinor representation of $\mathbf{16}_+$. The supercharges are also found to be in a $\mathbf{16}_+$ representation. Under the reduction of the $R^{9,1}$ theory to $R^{7,1}$, the global symmetry of the resulting 8-dimensional theory reduces

to

$$SO(9,1) \rightarrow SO(7,1) \times U(1)_R. \quad (1)$$

The 10-d gauge field \mathcal{A} decomposes into an 8-d gauge field A and two scalars $A_{8,9}$, combined into two complex fields

$$\varphi = A_8 + iA_9 \in (1, +1), \quad \bar{\varphi} = A_8 - iA_9 \in (1, -1)$$

which transform trivially under $SO(7,1)$ and have ± 1 charges under $U(1)_R$ in (1). Also, from the spinor decomposition we get two chiral fermions Ψ_{\pm} transforming as

$$\mathbf{16}_+ \rightarrow (S_+, \frac{1}{2}) + (S_-, -\frac{1}{2}).$$

Thereupon, the 8-d theory is compactified on a surface of two complex dimensions \mathcal{S} resulting in a four-dimensional field theory on $R^{7,1} \rightarrow R^{3,1} \times \mathcal{S}$, with reduced global symmetry dictated by the decomposition

$$SO(7,1) \times U(1)_R \rightarrow SO(3,1) \times SO(4) \times U(1)_R.$$

The 8-d spinor Ψ_+ decomposes as

$$\left(S_+, \frac{1}{2}\right) \rightarrow \left((2,1), (2,1), \frac{1}{2}\right) \oplus \left((1,2), (1,2), -\frac{1}{2}\right)$$

with respect to $SO(4) \times U(1)_R \sim SU(2) \times SU(2) \times U(1)_R$, and similarly for Ψ_- .

The compact surface \mathcal{S} is a nontrivial Riemannian four manifold and spinors (needed to define the 4-d supercharges) are globally not well defined. To preserve $\mathcal{N} = 1$ SUSY we embed $U(1)_R$ into the $U(2) \in SO(4)$. (Note that the Kähler structure of \mathcal{S} is preserved only by one $U(2)$, so spinors can have well defined properties only under the latter). Indeed, denoting J the generator of $U(1) \in U(2)$ and R that of $U(1)_R$, either of the combinations $J_{\pm} = J \pm 2R$ preserves one supersymmetry. For the Ψ_+ and the ϵ_+ generator choosing J_+ we have

$$\left(S_+, \frac{1}{2}\right) \rightarrow \{(2,1) \otimes 2_1\} \oplus \{(1,2) \otimes (1_2 \oplus 1_0)\}$$

and analogously for Ψ_- , ϵ_- . The fields descending from Ψ_+ decomposition are denoted as follows

$$\{(1,2) \otimes 1_0\} \rightarrow \bar{\eta}^{\dot{\alpha}}, \quad \{(2,1) \otimes 2_1\} \rightarrow \psi_m^{\alpha}, \quad \{(1,2) \otimes 1_2\} \rightarrow \bar{\chi}_{\bar{m}\bar{n}}^{\dot{\alpha}},$$

and constitute a zero, one and two form respectively,

$$\eta^{\dot{\alpha}}, \quad \psi^{\alpha} = \psi_m^{\alpha} d\bar{z}^{\bar{m}}, \quad \chi^{\dot{\alpha}} = \bar{\chi}_{\bar{m}\bar{n}}^{\dot{\alpha}} d\bar{z}^{\bar{m}} \wedge d\bar{z}^{\bar{n}}$$

and analogously for the conjugate Ψ_- .

Under $SO(3, 1) \times U(2) \times U(1)_{J_+}$ the scalars $\varphi, \bar{\varphi}$ transform as

$$\varphi = \{(1, 1) \otimes 1_{-2}\}, \quad \bar{\varphi} = \{(1, 1)_{+2}\}$$

while from the dimensional reduction of the 8-d vector A , we obtain the 4-d vector A_μ and the scalars $A_m(A_{\bar{m}})$ which transform as

$$A_{\bar{m}} = \{(1, 1) \otimes 2_{+1}\}.$$

The above fields pair up into one gauge multiplet and two $\mathcal{N} = 1$ chiral multiplets as follows:

$$(A_\mu, \eta^\alpha), (A_{\bar{m}}, \psi_{\bar{m}}^\alpha), (\varphi_{mn}, \chi_{mn}^\alpha). \quad (2)$$

3 KK-modes and the GUT scale

In F-theory, threshold corrections associated to KK-massive modes arise from gauge fields as well as from matter fields in the intersections. As already asserted in the introduction, KK-massive modes from the chiral and the Higgs sectors add up to a common shift of the gauge coupling constants at M_{GUT} . Indeed, we will prove that this happens when the charges q_i associated to the matter curves Σ_{q_i} are genuinely embedded into the function $\mathcal{T}(q_i)$ which defines the torsion. Thus, in this respect the F-theory case looks pretty much the same as in M-theory [18]. In the following, we will first give a brief account of the gauge thresholds computations adopting the techniques of [18] developed for G_2 -manifolds, while we will follow [10] for the case of F-theory we are interested in. Next, we will proceed with the computation of KK-thresholds from the chiral matter and the Higgs curves.

3.1 The gauge multiplet

We write the decomposition of the $SU(5)$ gauge multiplet under the SM symmetry as

$$24 \rightarrow R_0 + R_{-5/6} + R_{5/6}$$

with

$$R_0 = (8, 1)_0 + (1, 3)_0 + (1, 1)_0, \quad R_{-5/6} = (3, 2)_{-5/6}, \quad R_{5/6} = (\bar{3}, 2)_{5/6}. \quad (3)$$

Massless fields in the bulk are given by a topologically invariant quantity, the Euler characteristic \mathcal{X} , thus, in order to avoid the massless exotics $R_{\pm 5/6}$ we impose the condition $\mathcal{X}(S, L^{5/6}) = 0$. On the other hand, the massive modes in representations (3) induce threshold effects to the running of the gauge couplings. At the one-loop level we write

$$\frac{16\pi^2}{g_a^2(\mu)} = \frac{16\pi^2 k_a}{g_s^2} + b_a \log \frac{\Lambda^2}{\mu^2} + \mathcal{S}_a^{(g)}, \quad a = 3, 2, Y. \quad (4)$$

Here Λ is the gauge theory cutoff scale, $k_a = (1, 1, 5/3)$ are the normalization coefficients for the usual embedding of the Standard Model into $SU(5)$, g_s is the value of the gauge coupling at the high scale and $\mathcal{S}_a^{(g)}$ stand for the gauge fields thresholds. The one-loop β -function coefficients b_a for the massless spectrum (in the notation of [18]) are

$$b_a = 2 \text{Str}_{M=0} Q_a^2 \left(\frac{1}{12} - \chi^2 \right) \quad (5)$$

where χ is the helicity operator and Q_a stands for the three generators of the Standard Model gauge group $SU(3) \times SU(2) \times U(1)_Y$. In computing the supertrace Str we count bosonic contributions with weight $+1$ and fermionic with -1 . Similarly, the one-loop threshold corrections from the KK-massive modes in R_i are

$$\mathcal{S}_a^{(g)} = 2 \sum_i \text{Tr}_{R_i} Q_a^2 \text{Str}_{M \neq 0} \left(\frac{1}{12} - \chi^2 \right) \log \frac{\Lambda^2}{M^2}. \quad (6)$$

The squared masses of the KK modes in the threshold formula correspond to the massive spectrum of the Laplacian Δ_{k,R_i} acting on each k -form of the representation R_i . In the previous section we saw that the spectrum (2) consists of zero, one and two form multiplets. Each eigenvector of the zero-form Laplacian Δ_{0,R_i} contributes a vector multiplet with helicities $1, -1, \frac{1}{2}, -\frac{1}{2}$, while the one-form Laplacian Δ_{1,R_i} gives a chiral multiplet with helicities $0, 0, \frac{1}{2}, -\frac{1}{2}$. Finally, Δ_{2,R_i} is associated to anti-chiral multiplets. The sum of all the contributions to the gauge fields thresholds is

$$\mathcal{S}_a^{(g)} = 2 \sum_i \text{Tr}_{R_i} (Q_a^2) \mathcal{K}_i \quad (7)$$

with [10]

$$\mathcal{K}_i = \frac{3}{2} \log \det' \frac{\Delta_{0,R_i}}{\Lambda^2} - \frac{1}{2} \log \det' \frac{\Delta_{1,R_i}}{\Lambda^2} - \frac{1}{2} \log \det' \frac{\Delta_{2,R_i}}{\Lambda^2} \quad (8)$$

where the prime on \det' means that zero modes are omitted. Using the well known properties characterizing the massive spectra of the Laplacians Δ_{k,R_i} , it has been shown [10] that expression (8) is the Ray-Singer analytic torsion \mathcal{T}_i [17]; more precisely,

$$2\mathcal{T}_i = \mathcal{K}_i = 2 \log \det' \frac{\Delta_{0,R_i}}{\Lambda^2} - \log \det' \frac{\Delta_{1,R_i}}{\Lambda^2}. \quad (9)$$

Note that for the trivial representation R_0 there exist zero-modes and the torsion differs from \mathcal{K}_0 by a scaling dependent part $\propto 2 \log(V_S^{1/2} \Lambda^2)$ where V_S is the volume of the compact surface S . A detailed analysis on the scaling dependence can be found in [10]. Returning to (6) we compute the traces and use the fact that $\mathcal{K}_{5/6} = \mathcal{K}_{-5/6}$ to get

$$\left(\mathcal{S}_Y^{(g)}, \mathcal{S}_2^{(g)}, \mathcal{S}_3^{(g)} \right) = \left(\frac{50}{3} \mathcal{K}_{5/6}, 6\mathcal{K}_{5/6} + 4\mathcal{K}_0, 4\mathcal{K}_{5/6} + 6\mathcal{K}_0 \right). \quad (10)$$

Using the torsion \mathcal{T}_i and the β -functions $b_a^{(g)} = (0, -6, -9)$, we can rewrite the above as

$$\mathcal{S}_a^{(g)} = \frac{4}{3} b_a^{(g)} (\mathcal{T}_{5/6} - \mathcal{T}_0) + 20 k_a \mathcal{T}_{5/6}. \quad (11)$$

Absorbing the term proportional to k_a into a redefinition of g_s we may now write the one loop equation (4) for the running of the gauge couplings [14] as

$$\frac{16\pi^2}{g_a^2(\mu)} = \left(\frac{16\pi^2}{g_s^2} + 20 \mathcal{T}_{5/6} \right) k_a + b_a^{(g)} \log \frac{\exp [4/3 (\mathcal{T}_{5/6} - \mathcal{T}_0)]}{\mu^2 V_S^{1/2}}. \quad (12)$$

The form (12) suggests that we can define M_{GUT} as [14]

$$M_{GUT}^2 = \frac{\exp [4/3 (\mathcal{T}_{5/6} - \mathcal{T}_0)]}{V_S^{1/2}} \quad (13)$$

and a gauge coupling g_U at the GUT scale shifted by

$$\frac{16\pi^2}{g_U^2} = \frac{16\pi^2}{g_s^2} + 20 \mathcal{T}_{5/6}. \quad (14)$$

Furthermore, if we associate the world volume factor $V_S^{-1/4}$ with the characteristic F-theory compactification scale M_C , we can write this equation as follows

$$M_{GUT} = e^{2/3(\mathcal{T}_{5/6} - \mathcal{T}_0)} M_C. \quad (15)$$

Thus, as far as the gauge fields thresholds are concerned, we find that M_{GUT} is given in terms of M_C through an elegant relation. In the next section we will present the matter fields contributions and investigate the conditions under which this relation continues to hold true.

3.2 The chiral matter

Here, we will discuss contributions arising from chiral matter, the Higgs fields and the possible exotic representations. In F-theory constructions, these fields arise in the intersections of the GUT-brane with other 7-branes as well as from the decomposition of the adjoint representation in the bulk. We have already imposed the conditions which avoid the exotic bulk zero modes $R_{-5/6} = (3, 2)_{-5/6}$ and $R_{5/6} = (\bar{3}, 2)_{5/6}$, so we are only left with light matter fields at the intersections. In the $SU(5)$ case, these correspond to the standard $10, \bar{10}$ and $5, \bar{5}$ non-trivial representations and contribute to the RG running a term of the form $b_a^x \log \Lambda'^2 / \mu^2$ where b_a^x are the β -function coefficients for the matter fields, and Λ' a cutoff scale which may differ from the gauge cutoff Λ .

We should mention that the $U(1)_Y$ -flux introduced in order to break $SU(5)$ might eventually lead to incomplete $SU(5)$ representations, spoiling thus the gauge coupling unification. However, it is still possible to work out realistic cases [19, 20, 14] where the matter fields add up to complete $SU(5)$ multiplets, so that the b_a^x -functions contribute in proportion to the coefficients k_a . Then, as in the case of the gauge contributions discussed earlier, we can absorb the logarithmic Λ' -dependence into a redefinition of the gauge coupling. Nevertheless, the color triplet pair descending from the $5_H + \bar{5}_H$

Thresholds	$SU(3)$	$SU(2)$	$U(1)$
$S_a^{\bar{5}}$	$\mathcal{K}_{1/3}$	$\mathcal{K}_{-1/2}$	$\mathcal{K}_{-1/2} + 2/3 \mathcal{K}_{1/3}$
S_a^{10}	$2\mathcal{K}_{1/6} + \mathcal{K}_{-2/3}$	$3\mathcal{K}_{1/6}$	$1/3\mathcal{K}_{1/6} + 2\mathcal{K}_1 + 8/3\mathcal{K}_{-2/3}$

Table 1: Threshold corrections $S_a^{\bar{5}}, S_a^{10}$ to the three gauge couplings from Kaluza-Klein massive modes along the matter curves.

Higgs quintuplets must receive a mass at a relatively high scale $M_X \leq M_{GUT}$ so to avoid rapid proton decay. Taking all into account, we write (12) in the form

$$\frac{16\pi^2}{g_a^2(\mu)} = k_a \frac{16\pi^2}{g_{GUT}^2} + (b_a^{(g)} + b_a) \log \frac{M_{GUT}^2}{\mu^2} + b_a^T \log \frac{M_{GUT}^2}{M_X^2} \quad (16)$$

where we have split $b_a^x = b_a + b_a^T$ with b_a denoting the MSSM β -functions and b_a^T the color triplet part.

In the context of F-theory constructions, in addition to the light degrees of freedom on matter curves, one also has to include contributions from Kaluza-Klein massive modes. As already explained, this is in contrast to the case of G_2 manifolds, where no new contributions are introduced to the gauge coupling running apart from the massless states [18]. Threshold contributions arise from the massive states along the $\Sigma_{\bar{5}}$ and Σ_{10} matter curves. To compute them we write down the decompositions of the corresponding representations

$$10 \rightarrow (3, 2)_{\frac{1}{6}} + (\bar{3}, 1)_{-\frac{2}{3}} + (1, 1)_1, \quad \bar{5} \rightarrow (\bar{3}, 1)_{\frac{1}{3}} + (1, 2)_{-\frac{1}{2}}.$$

For each of the above matter curves we consider the Laplacian acting on the representations with eigenvalues corresponding to chiral and anti-chiral fields. Thus, for the massive modes of Σ_{10} we have

$$\mathcal{K}_{\Sigma_{10}} = -\frac{1}{2} \log \det' \frac{\Delta_{0,Y}}{\Lambda'^2} - \frac{1}{2} \log \det' \frac{\Delta_{1,Y}}{\Lambda'^2}$$

and similarly for the $\Sigma_{\bar{5}}$. Denoting by $S_{a=3,2,Y}$ the thresholds to the three gauge factors of the SM, for a representation r we then have

$$S_a^r = \sum_i 2\text{Tr}(Q_{a,r}^2) \mathcal{K}_i.$$

Computing the traces we readily find the KK-thresholds shown in Table 1.

We will now further elaborate on the form of the corrections, and attempt to recast them as a sum of two different pieces, one being proportional to k_a . The KK-thresholds induced by the $\bar{5}$ can be written as follows;

$$S_a^{\bar{5}} = -\frac{2}{3} \beta_a^{\bar{5}} (\mathcal{K}_{-1/2} - \mathcal{K}_{1/3}) + k_a \cdot (\mathcal{K}_{-1/2}) \quad (17)$$

where we have introduced the “ β ”-coefficients

$$\beta_{3,2,1}^{\bar{5}} = \{\frac{3}{2}, 0, 1\}$$

and, as usually, $k_a = (1, 1, 5/3)$. For the Σ_{10} we can write the thresholds related to $U(1)_Y$ in the form

$$\begin{aligned} S_1^{10} &= \frac{1}{3} \mathcal{K}_{1/6} + \frac{8}{3} \mathcal{K}_{-2/3} + 2\mathcal{K}_1 \\ &= \frac{8}{3} (\mathcal{K}_{-2/3} - \mathcal{K}_{1/6}) - 2 (\mathcal{K}_{1/6} - \mathcal{K}_1) + \frac{15}{3} \mathcal{K}_{1/6}. \end{aligned} \quad (18)$$

We observe that in the two parentheses the $U(1)_Y$ charge differences obey the relation $q_i - q_j = -\frac{5}{6}$. This suggests that a non-trivial line bundle structure could be sought with the ‘periodicity’ property $\mathcal{K}_{q_i} - \mathcal{K}_{q_j} = f(q_i - q_j)$ so that

$$\mathcal{K}_{1/6} - \mathcal{K}_1 = \mathcal{K}_{-2/3} - \mathcal{K}_{1/6}.$$

Adopting this assumption, we finally get

$$\begin{aligned} S_1^{10} &= \frac{2}{3} (\mathcal{K}_{-2/3} - \mathcal{K}_{1/6}) + \frac{5}{3} \cdot (3 \mathcal{K}_{1/6}) \\ S_2^{10} &= 0 (\mathcal{K}_{-2/3} - \mathcal{K}_{1/6}) + 1 \cdot (3 \mathcal{K}_{1/6}) \\ S_3^{10} &= 1 (\mathcal{K}_{-2/3} - \mathcal{K}_{1/6}) + 1 \cdot (3 \mathcal{K}_{1/6}). \end{aligned}$$

These relations can be written in compact form in straight analogy with (17) as

$$S_a^{10} = \frac{2}{3} \beta_a^{10} (\mathcal{K}_{-2/3} - \mathcal{K}_{1/6}) + k_a \cdot (3 \mathcal{K}_{1/6})$$

with $\beta_a^{10} = \beta_a^{\bar{5}}$.

Recalling the Ray-Singer torsion \mathcal{T}_i we may write threshold terms for both matter curves as follows

$$S_a^{\bar{5}} = -\frac{4}{3} \beta_a^{\bar{5}} (\mathcal{T}_{-1/2} - \mathcal{T}_{1/3}) + k_a (2 \cdot \mathcal{T}_{-1/2}) \quad (19)$$

$$S_a^{10} = +\frac{4}{3} \beta_a^{10} (\mathcal{T}_{-2/3} - \mathcal{T}_{1/6}) + k_a (6 \cdot \mathcal{T}_{1/6}). \quad (20)$$

We now observe that the hypercharge assignments in both Σ_{10} and $\Sigma_{\bar{5}}$ satisfy the same condition $q_i - q_j = -\frac{5}{6}$. Given this property and the fact that the torsion is a topologically invariant quantity, one could assume the existence of bundle structures for Σ_{10} and $\Sigma_{\bar{5}}$ matter curves characterized by the same topological properties so that we may envisage a specific embedding of the hypercharge generator implying

$$\mathcal{T}_{-1/2} - \mathcal{T}_{1/3} = \mathcal{T}_{-2/3} - \mathcal{T}_{1/6} = 0. \quad (21)$$

In this limit, threshold contributions which are not proportional to k_a cancel in both Σ_{10} and $\Sigma_{\bar{5}}$ curves.

In general, matter curves accommodating different representations of the gauge group do not necessarily bear the same bundle structure. In particular, in the case of $SU(5)$ it often happens that the $\Sigma_{\bar{5}}$ curve is of higher genus than the Σ_{10} for example. One of course could not exclude the possibility that the condition (21) can be separately satisfied for surfaces of different genera. However, we mention that in the recent literature one can find several examples where Σ_{10} and $\Sigma_{\bar{5}}$ curves are of the same genus and the required property holds true. To give further support to our argument, we will briefly present a model discussed in ref [8]. Bearing in mind that in order to decouple gauge dynamics from gravity and allow for the possibility $M_{GUT} \ll M_{Planck}$, we choose the surface S to be one of the del Pezzo type dP_n with $n = 1, 2, \dots, 8$. We choose dP_8 which is generated by the hyperplane divisor H from \mathbb{P}^2 and the exceptional divisors $E_{1,\dots,8}$ with intersection numbers

$$H \cdot H = 1, \quad H \cdot E_i = 0, \quad E_i \cdot E_j = -\delta_{ij}. \quad (22)$$

We also note that the canonical divisor for dP_8 is

$$K_S = -c_1(dP_8) = -3H + \sum_{i=1}^8 E_i. \quad (23)$$

Then, denoting with C and g the class and the genus of a matter curve respectively, we have $C \cdot (C + K_S) = 2g - 2$. In the particular example of section 17 in ref [8] the 10_M chiral matter of the three generations resides on one Σ_{10} , with $C = 2H - E_1 - E_5$ and the three $\bar{5}_M$ on a single $\Sigma_{\bar{5}}^1$ curve with $C = H$. Higgs fields 5_H and $\bar{5}_{\bar{H}}$ are localized on different $\Sigma_{\bar{5}}^{2,3}$ matter curves with classes $C = H - E_1 - E_3$ and $H - E_2 - E_4$ respectively. Checking the relevant intersections, one readily finds that all the above matter curves are of the same genus $g = 0$ and therefore the criterion is fulfilled.

Returning to the threshold contributions (19,20), once the parts proportional to $\beta_a^{\bar{5}}, \beta_a^{10}$ cancel out we observe that the remaining contributions from KK thresholds are just those proportional to the coefficients k_a and consequently, they only induce a shift of the gauge coupling value at M_{GUT} . We finally get

$$\frac{16\pi^2}{g_a^2(\mu)} = \left(\frac{16\pi^2}{g_s^2} + 20\mathcal{T}_{5/6} + 6\mathcal{T}_{1/6} + 2\mathcal{T}_{1/3} \right) k_a + (b_a^{(g)} + b_a) \log \frac{M_{GUT}^2}{\mu^2} + b_a^T \log \frac{M_{GUT}^2}{M_X^2}. \quad (24)$$

Thus, matter thresholds leave the GUT scale M_{GUT} intact, their only net effect amounts to a further shift of the common gauge coupling. The value of the latter at the GUT scale is defined by

$$\frac{16\pi^2}{g_{GUT}^2} = \frac{16\pi^2}{g_s^2} + 20\mathcal{T}_{5/6} + 6\mathcal{T}_{1/6} + 2\mathcal{T}_{1/3}. \quad (25)$$

Note in passing that in the case where KK-modes from the gauge multiplet are associated to a bundle with different properties, we denote $\mathcal{T}_{5/6} \rightarrow \mathcal{T}'_{5/6}$ while the above analysis still holds.

We observe that (24) are just the one-loop renormalization group equations for the minimal $SU(5)$ GUT, with extra color triplets becoming massive at a scale $M_X \leq M_{GUT}$. We further note that in F -theory constructions, a $U(1)_Y$ flux mechanism is employed to break the $SU(5)$ symmetry, inducing a splitting of the gauge couplings at the GUT scale. Interestingly, this gauge coupling splitting is still consistent with a unification scale $M_{GUT} \sim 2 \times 10^{16}$ GeV provided that the triplets receive a mass at a scale determined by consistency conditions [11, 13].

3.2.1 Example: The case of non-trivial line bundle

In this section we will present an example of Σ_{10}, Σ_5 matter curves with non-trivial structure. In particular, we will consider the case of genus $g = 1$ Riemann surfaces and use the torsion results of [17] to compute the KK-matter contributions. We are interested in the masses of the KK modes, that is the eigenvalues of the Laplacian on a complex one dimensional Riemann surface. Thresholds from these KK-massive modes are given as functions of the torsion which is expressed in terms of the eigenvalues through the zeta function associated to the Laplacian Δ_k

$$\Delta_{k,R(V)} = (\bar{\partial} + \bar{\partial}^\dagger)^2 = \bar{\partial}\bar{\partial}^\dagger + \bar{\partial}^\dagger\bar{\partial}. \quad (26)$$

If we collectively denote ψ_k^n as the k -form eigenfunction, then

$$\Delta_{k,R(V)}\psi_k^n = \lambda_n^k \psi_k^n \quad (27)$$

where λ_n^k is the corresponding eigenvalue which in four dimensions corresponds to a mass squared. The associated zeta-function is given by

$$\zeta_{\Delta_k}(s) = \sum_n \frac{1}{\lambda_n^s} = \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} \text{Tr} (e^{-\Delta_k t}) dt \quad (28)$$

so that

$$\ln(\text{Det} \Delta_k) = - \left. \frac{d\zeta_{\Delta_k}(s)}{ds} \right|_{s=0}.$$

The torsion is written as

$$\mathcal{T} = \sum_k (-1)^{k+1} k \left. \frac{d\zeta_{\Delta_k}(s)}{ds} \right|_{s=0}. \quad (29)$$

For our application, we have already assumed a Riemann surface of genus $g = 1$ and a character given by $\chi = \exp\{2\pi i(mu + nv)\}$ with the identification $\chi \leftrightarrow u - \tau v$. The eigenvalues are

$$\lambda_n = \frac{4\pi^2}{\text{Im}\tau} |u + m - \tau(v + n)|^2. \quad (30)$$

The eigenfunctions are

$$\psi_n = \exp \left\{ \frac{2\pi i}{\text{Im}\tau} \text{Im}[z(u + m - \bar{\tau}(v + n))] \right\}.$$

Given the eigenvalues (30), the torsion can be computed [17] using (29) and (28). Because of its central role in this example, we present the basic steps of its derivation, adapting the notation [17] into our formalism. Let us assume that $\tau = \tau_1 + i\tau_2$ and let us define $S_1 = \text{Tr} (e^{-\Delta_k t})$ which amounts to the calculation of the following double sum:

$$S_1 = \sum_{m,n=-\infty}^{\infty} \exp \left[-\frac{4\pi^2 t}{\tau_2^2} \left((u+m)^2 + \tau^2 (v+n)^2 - 2\tau_1 (u+m)(v+n) \right) \right]. \quad (31)$$

Applying the Poisson summation formula we get

$$S_1 = \frac{\tau_2}{4\pi t} \sum_{m,n=-\infty}^{\infty} \exp \left[-\frac{1}{4t} (m^2 \tau^2 + n^2 + 2\tau_1 mn) + 2\pi i (mu + nv) \right]. \quad (32)$$

Putting $a = (m^2 \tau^2 + n^2 + 2\tau_1 mn)$ and substituting into (28), we get

$$\zeta(s) = \frac{\tau_2}{4\pi} \frac{1}{\Gamma(s)} \sum_{m,n=-\infty}^{\infty} \int_0^{\infty} dt \, t^{s-2} e^{-\frac{a}{4t}} \exp [2\pi i (mu + nv)]. \quad (33)$$

For $s > 1$ the integration gives

$$\zeta(s) = \frac{\tau_2}{4\pi} \frac{\Gamma(1-s)}{\Gamma(s)} \sum_{m,n=-\infty}^{\infty} \left(\frac{4}{a} \right)^{1-s} \exp (2\pi i (mu + nv)). \quad (34)$$

We readily now find that

$$\zeta'(0) = \frac{\tau_2}{\pi} \sum_{m,n=-\infty}^{\infty} \frac{\exp [2\pi i (mu + nv)]}{(m^2 \tau^2 + n^2 + 2\tau_1 mn)}. \quad (35)$$

According to Kronecker's second limit theorem, the singular term $m = 0, n = 0$ has to be omitted [21]. This way we get

$$\zeta'(0) = \frac{\tau_2}{\pi} \sum_{n \neq 0} \frac{\exp [2\pi i nv]}{n^2} + \frac{\tau_2}{\pi} \sum_{m \neq 0} e^{2\pi i mu} \sum_{n=-\infty}^{\infty} \frac{e^{2\pi i nv}}{m^2 \tau^2 + n^2 + 2\tau_1 mn}. \quad (36)$$

The first sum is [22]

$$\sum_{n \neq 0} \frac{\exp [2\pi i nv]}{n^2} = 2 \sum_{n=1}^{\infty} \frac{\cos 2\pi vn}{n^2} = \frac{3(2\pi v)^2 - 6\pi(2\pi v) + 2\pi^2}{6} = 2\pi^2 \left(v^2 - v + \frac{1}{6} \right)$$

where $0 < v < 1$. The n sum in the second term of (36) can be evaluated by means of the Poisson formula

$$\sum_{n=-\infty}^{\infty} f(-n) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{2\pi i nx} f(x) dx \quad (37)$$

where we take

$$f(x) = \frac{e^{2i\pi vx}}{m^2\tau^2 + x^2 + 2\tau_1 mx}. \quad (38)$$

The denominator can be written as

$$m^2\tau^2 + x^2 + 2\tau_1 mx = (m\tau_1 + x)^2 + m^2\tau_2^2 \quad (39)$$

so that

$$\begin{aligned} I &= \int_{-\infty}^{\infty} dx \frac{e^{2i\pi(n+v)x}}{(m\tau_1 + x)^2 + m^2\tau_2^2} = \int_{-\infty}^{\infty} dx \frac{e^{-2i\pi(n+v)m\tau_1} e^{2i\pi(n+v)x}}{x^2 + m^2\tau_2^2} \\ &= \pi \frac{e^{-2i\pi(n+v)m\tau_1} e^{-2\pi|v+n||m\tau_2|}}{|m\tau_2|}. \end{aligned} \quad (40)$$

Restricting to the upper plane so that $\tau_2 = \text{Im } \tau > 0$, we finally get

$$\zeta'(0) = 2\pi\tau_2 \left(v^2 - v + \frac{1}{6} \right) + \sum_{n=-\infty}^{\infty} \sum_{m \neq 0} \frac{1}{|m|} e^{-2|m||v+n|\pi\tau_2 - 2i\pi(n+v)m\tau_1 + 2i\pi mu}.$$

The sum over m gives

$$\sum_{m \neq 0} \frac{1}{|m|} e^{-2a\pi|m| + 2i\pi bm} = -\ln \left(1 - e^{-2\pi(a+bi)} \right) - \ln \left(1 - e^{-2\pi(a-bi)} \right) \quad (41)$$

or

$$\zeta'(0) = 2\pi\tau_2 \left(v^2 - v + \frac{1}{6} \right) - \sum_{n=-\infty}^{\infty} \ln \left| 1 - e^{-2|v+n|\pi\tau_2 + 2i\pi(n+v)\tau_1 - 2i\pi u} \right|^2.$$

Consider now the exponent

$$2i\pi [|v+n| i\tau_2 + (n+v)\tau_1 - u]. \quad (42)$$

For $n = 0$ the terms inside the bracket become $-(u - \tau v)$ while for $n > 1$ we get

$$(v + |n|) i\tau_2 + (|n| + v)\tau_1 - u = |n|\tau - (u - \tau v). \quad (43)$$

For $n < -1$ we get

$$(|n| - v) i\tau_2 + (-|n| + v)\tau_1 - u = -|n|\tau^* - (u - v\tau^*) = [2i\pi(|n|\tau + (u - \tau v))]^*.$$

All the above cases can be represented in a compact form as follows:

$$\zeta'(0) = 2\pi\tau_2 \left(v^2 - v + \frac{1}{6} \right) - \sum_{n=-\infty}^{\infty} \ln \left| 1 - e^{2i\pi(|n|\tau - \varepsilon_n(u - \tau v))} \right|^2$$

where we have introduced the sign convention

$$\varepsilon_n = \text{sign} \left(n + \frac{1}{2} \right). \quad (44)$$

Now consider the function

$$g(w, \tau) = \prod_{n=-\infty}^{\infty} (1 - \exp[2i\pi(|n|\tau - \varepsilon_n w)]). \quad (45)$$

Separating out the zero mode we may write

$$g(w, \tau) = (1 - \exp[-2i\pi w]) \prod_{n=1}^{\infty} (1 - \exp[2i\pi(n\tau - w)]) \prod_{n=1}^{\infty} (1 - \exp[2i\pi(n\tau + w)]). \quad (46)$$

Using the nome $q = e^{i\pi\tau}$ we get

$$g(w, \tau) = 2i \sin \pi w e^{-i\pi w} \prod_{n=1}^{\infty} (1 - 2q^{2n} \cos 2\pi w + q^{4n}). \quad (47)$$

The elliptic function ϑ_1 is defined as

$$\vartheta_1(w, \tau) = 2q^{\frac{1}{4}} \sin \pi w \prod_{n=1}^{\infty} (1 - 2q^{2n} \cos 2\pi w + q^{4n}) (1 - q^{2n}). \quad (48)$$

Using the Dedekind eta function

$$\eta(\tau) = q^{\frac{1}{12}} \prod_{n=1}^{\infty} (1 - q^{2n}) \quad (49)$$

we deduce that

$$\vartheta_1(w, \tau) = -iq^{\frac{1}{6}} e^{i\pi w} \eta(\tau) g(w, \tau) \quad (50)$$

or

$$\vartheta_1(w, \tau) = -ie^{i\pi(w + \frac{\tau}{6})} \eta(\tau) g(w, \tau). \quad (51)$$

This way,

$$\begin{aligned} \sum_{n=-\infty}^{\infty} \ln \left| 1 - e^{2i\pi(|n|\tau - \varepsilon_n(u - \tau v))} \right|^2 &= \ln \left| \frac{\vartheta_1(u - \tau v, \tau)}{\eta(\tau)} \right|^2 \\ &\quad + \ln \left(e^{-i\pi(u - \tau(v - \frac{1}{6}))} e^{i\pi(u - \tau^*(v - \frac{1}{6}))} \right) \\ &= 2 \ln \left| \frac{\vartheta_1(u - \tau v, \tau)}{\eta(\tau)} \right| + \ln \left(e^{-2\pi\tau_2(v - \frac{1}{6})} \right) \\ &= 2 \ln \left| \frac{\vartheta_1(u - \tau v, \tau)}{\eta(\tau)} \right| - 2\pi\tau_2 \left(v - \frac{1}{6} \right). \end{aligned} \quad (52)$$

Finally, collecting all the terms we get

$$\begin{aligned}\zeta'(0) &= 2\pi\tau_2 \left(v^2 - v + \frac{1}{6} \right) - 2 \ln \left| \frac{\vartheta_1(u - \tau v, \tau)}{\eta(\tau)} \right| + 2\pi\tau_2 \left(v - \frac{1}{6} \right) \\ &= -2 \ln \left| e^{i\pi\tau v^2} \frac{\vartheta_1(u - \tau v, \tau)}{\eta(\tau)} \right|\end{aligned}\quad (53)$$

Therefore, the analytic torsion is

$$\mathcal{T}_z = \ln \left| \frac{e^{\pi i v^2 \tau} \vartheta_1(z, \tau)}{\eta(\tau)} \right|, \quad z = u - \tau v. \quad (54)$$

In order to use this result, we need to make a proper identification of the hypercharge q_i . Let us first recall the following identity for $\vartheta_1(z, \tau)$:

$$\vartheta_1(z + \tau, \tau) = -e^{-\pi i \tau} e^{-2\pi i z} \vartheta_1(z, \tau). \quad (55)$$

For $z = u - \tau v$ this becomes

$$\vartheta_1(u - \tau v + \tau, \tau) = -e^{\pi i (2v-1)} e^{-2\pi i u} \vartheta_1(u - \tau v). \quad (56)$$

In terms of the variables u, v , we observe that the transformation is essentially equivalent to the shift $v \rightarrow v-1$, i.e. the left part can be rewritten as $\vartheta_1(u - \tau(v-1), \tau)$. Consequently, for two different points $v, v-1$ the torsion reads

$$\mathcal{T}_v \equiv \mathcal{T}_{z=u-\tau v} = \ln \left| \frac{e^{\pi i \tau v^2} \vartheta_1(u - \tau v, \tau)}{\eta(\tau)} \right| \quad (57)$$

$$\mathcal{T}_{v-1} \equiv \mathcal{T}_{z=u-\tau(v-1)} = \ln \left| \frac{e^{\pi i \tau (v-1)^2} \vartheta_1(u - \tau(v-1), \tau)}{\eta(\tau)} \right|. \quad (58)$$

Using the identity (56) the numerator in the logarithmic quantity (58) becomes

$$\begin{aligned}e^{\pi i \tau (v-1)^2} \vartheta_1(u - \tau(v-1), \tau) &= -e^{\pi i \tau (v-1)^2} e^{\pi i \tau (2v-1)} e^{-2\pi i u} \vartheta_1(u - \tau v) \\ &= -e^{-2\pi i u} e^{\pi i \tau v^2} \vartheta_1(u - \tau v, \tau).\end{aligned}\quad (59)$$

Now, substituting into the torsion formula and taking into account that u is real, we obtain

$$\begin{aligned}\mathcal{T}_{z=u-\tau(v-1)} &= \ln \left| -e^{-2\pi i u} e^{\pi i \tau v^2} \vartheta_1(u - \tau v, \tau) \right| \\ &= \ln \left| e^{\pi i \tau v^2} \vartheta_1(u - \tau v, \tau) \right| = \mathcal{T}_{z=u-\tau v}.\end{aligned}\quad (60)$$

Considering now two successive hypercharge values q_i, q_j such that $|q_i - q_j| = \frac{5}{6}$ and using the association

$$v_i = \frac{q_i}{|q_i - q_j|} \quad (61)$$

we get the identification

$$\mathcal{T}_{u-\tau v_i} \leftrightarrow \mathcal{T}_{q_i}.$$

With this embedding we can easily see that the differences $\mathcal{T}_{-2/3} - \mathcal{T}_{1/6}$ and $\mathcal{T}_{-1/2} - \mathcal{T}_{1/3}$ vanish and the result (24) is readily obtained.

We stress that this example, although not fully realistic (since we have restricted our investigation to the flat torus) is sufficient to support the aforementioned ideas. In proposing the above identification we relied on the assumption that a $U(1)$ symmetry is naturally associated with the one cycle of the torus, while the hypercharge identification seems to be in accordance with the notion of $U(1)$ fluxes piercing the matter curves. Indeed, we know that when the $U(1)$ fluxes are turned on they affect the multiplicity of the various massless representations along the matter curves. For example, assuming the $\Sigma_{\bar{5}}$ matter curve, the number of 5's and/or $\bar{5}$'s is determined by the fluxes of $U(1)_i$'s corresponding to some Cartan generators of the commutant gauge group inside E_8 (here being $SU(5)_{\perp}$). Furthermore, $U(1)_Y \in SU(5)_{GUT}$ determines in a similar manner the splitting of the standard model representations obtained from the decomposition of 10 and $\bar{5}$'s. Indeed, in the presence of $U(1)_Y \in SU(5)_{GUT}$ flux, we can express for example the splitting of the massless spectrum for n units of hyperflux for $5 \rightarrow (3, 1)_{1/3} + (1, 2)_{-1/2}$ as $\#(3, 1)_{1/3} - \#(1, 2)_{-1/2} = (v_d - v_l)n = n$. We notice that eq. (30) and the hypercharge association assumed in (61) imply also the same v -dependence of the corresponding massive modes.

3.2.2 On matter curves with higher genera

In the previous sections we have presented simple examples where threshold corrections from KK states associated to genus one matter curves do not alter the unification scale. For $g = 1$ the properties of the determinants are well understood and (at least in the case of flat torus) we can corroborate our assumption for the $U(1)_Y$ embedding with an explicit computation. However, in F-theory, we deal quite often with examples involving matter curves of higher genera ($g \geq 2$). In this case a natural extension of the $\bar{\partial}$ -torsion can be possibly related to the Selberg's zeta function [23]. Then one has to deal with the rather non-trivial task of seeking specific realistic cases where the required properties mentioned in the previous sections are satisfied. To convey an idea of the issues in this general case, we will give a brief account on the possibility of implementing our analysis for $g > 1$, leaving a more detailed consideration for future work.

To start with, we first note that the compact Riemannian manifold (for $g > 1$) can be written as \mathcal{H}/Γ , that is, it can be identified as the quotient of the upper half plane \mathcal{H} by the group of isometries Γ of \mathcal{H} with elements

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma : \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} z = \frac{az + b}{cz + d}$$

with the condition $|a + d| > 2$ ³. An element $\gamma \in \Gamma$ is called primitive if it is not a

³This is a space with hyperbolic geometry with metric $ds^2 = y^{-2}(dx^2 + dy^2)$ and constant negative

power of some other element in Γ . An element γ' is said to be conjugate to another γ if there exists an element γ_1 in Γ such that

$$\gamma' = \gamma_1 \gamma \gamma_1^{-1}$$

We denote $\{\gamma\}$ the set of elements which are conjugate to γ . This way, Γ is the union of disjoint conjugacy classes. If γ_0 is the primitive element of $\{\gamma\}$, then any other element in the same class can be written as $\gamma = \gamma_0^n$ for some integer power n . We mention that for a compact manifold the element $\gamma \in \Gamma$ can also be written as

$$\gamma \in \Gamma : \frac{z' - z_0}{z' - z_1} = e^{2\rho_\gamma} \frac{z - z_0}{z - z_1}$$

for two real fixed points $z_{0,1}$ and $\rho_\gamma > 0$. For given finite unitary representation $\chi(\gamma)$, the Selberg zeta-function is defined [17] as

$$Z(s, \chi) = \prod_{\{\gamma\}} \prod_{n=0}^{\infty} \det \left(1 - \chi(\gamma) e^{-\rho_\gamma(s+n)} \right) \quad (62)$$

with $\text{Re}(s) > 1$. Hence, any required properties of the torsion could be investigated with respect to its relation to the Selberg zeta function given by the general formula (62). For example, for two non-trivial unitary representations $\chi(\gamma)$ and $\chi'(\gamma')$ of Γ and for a compact Riemann surface of $g > 1$, according to a theorem by Ray and Singer [17] the difference $\ln(\mathcal{T}_0(\chi)) - \ln(\mathcal{T}_0(\chi'))$ is proportional to $\ln(Z(\chi) - \ln(Z(\chi')))$. Several studies [24, 25, 26, 27, 28] have revealed interesting properties of Selberg's function. It is envisaged that one can find examples where the required quantities exhibit periodicity properties and an appropriate hypercharge embedding could also be feasible. We plan to return to these issues in a future publication.

4 Conclusions

In unified theories emerging in the context of F-theory compactification, threshold corrections from Kaluza-Klein massive modes play a decisive role in gauge coupling unification and the determination of the GUT scale. In this work, we have revisited this issue in the context of a specific minimal unification scenario, the F-theory $SU(5)$ GUT. Although the problem of KK thresholds is in general quite complicated, in the model under consideration it gets remarkably simplified using the fact that these thresholds can be expressed in terms of a topologically invariant quantity, the Ray-Singer analytic torsion. Previous considerations have shown that the KK-modes from the gauge multiplets can be absorbed into a redefinition of the effective GUT mass scale and the string gauge coupling. However, the situation concerning KK-mode contributions emerging from the matter curves is less clear. Here, we have pursued this issue one step further, and analyzed the conditions to be imposed on the matter spectrum and the nature of

curvature $R = -1$.

bundle structure where matter resides, in order to ensure that the emerging F-theory GUT comply with low energy phenomenological expectations. We have given examples where matter resides on genus one matter curves with chiral matter forming complete $SU(5)$ multiplets, which are consistent with the minimal unification scenario. These models are also capable of reproducing the expected low energy values for the weak mixing angle and the strong gauge coupling. A short discussion is also devoted to the prospects of models possessing matter curves of higher genera.

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